

Transformations

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Lesson Title: Transformations! What is the Location?

Subject: Mathematics – transformational geometry

Grade: 7 or 8

Arkansas Student Learning Expectations:

G.9.7.2 Perform translations and reflections of two-dimensional figures using a variety of methods (paper folding, tracing, graph paper).

G.9.8.1 Determine a transformation's line of symmetry and compare the properties of the figure and its transformation.

G.9.8.2 Draw the results of translations and reflections about the x- and y-axis and the rotations of objects about the origin.

Description of Lesson:

Time: three 50 minute class periods (more depending on the level of the class).

Materials: (For each student) Transformation handout, glue stick, miras, straw (coffee stirrer works best), paddy paper or tracing paper cut to the size of the coordinate grid, 10 copies of each of the following polygons in 4 different colors- rectangle ABCD, isosceles triangle EFG, trapezoid HIJK, isosceles right triangle LMN (for the class) tape, scissors, overhead transparencies of the handout and the triangles (optional).

Preplanning: Prepare handout, send home figures to be cut out by the students, and gather other materials.

Engage: Warm-up: Ask "What is a transformation?" Discuss the term and relate this term to things the students may be familiar with "Flips, Slides and Turns." Define "Transformation."

Explore: Have the student glue down one of the ABCD rectangles and note this as the original or pre-image of the figure. Have one group glue theirs down in the 1st quadrant (at the same location), the second group glue theirs down in the 2nd quadrant, the third group glue down in the 3rd quadrant, the fourth group in the 4th quadrant, the 5th group glue theirs down on the y-axis (where part of their figure is in the 1st and 2nd or the 3rd and 4th quadrants) and the 6th group glue theirs down where the figure is on either side of the x-axis. Have each group note their original locations. *Note that the activity will be less difficult if the students make sure that the vertices of the figures are on coordinates that have whole numbers.* They will continue to use this location for the original in each

transformation. Have the students record the coordinates of the vertices in their Transformation handout.

Reflections: Have the students use miras to reflect the rectangle across the x-axis. When the students find the reflection, they need to glue a copy of the rectangle down and write down where the corresponding vertices are on the coordinate grid (A' , B' , C' and D').

Ask several students to state an original coordinate and its corresponding reflection point. Make a list on the board for the class to see. Ask the class if they can see a pattern? If you give them a coordinate can they tell you the reflection coordinate?

Finally ask them to give you a rule: **For any point (a,b) the reflection of it over the x-axis is $(a, -b)$.** Read as “**a opposite of b**”.

Have the students use miras to reflect the rectangle across the y-axis. When the students find the reflection, they need to glue a copy of the rectangle down and write down where the corresponding vertices are on the coordinate grid (A'' , B'' , C'' and D'').

Ask several students to state an original coordinate and its corresponding reflection point. Make a list on the board for the class to see. Ask the class if they can see a pattern? If you give them a coordinate can they tell you the reflection coordinate?

Finally ask them to give you a rule: **For any point (a,b) the reflection of it over the y-axis is $(-a, b)$.** Read as “**opposite of a, b**”.

Have the students use miras to reflect the rectangle across the line $y = x$. When the students find the reflection, they need to glue a copy of the rectangle down and write down where the corresponding vertices are on the coordinate grid (A''' , B''' , C''' and D''').

Ask several students to state an original coordinate and its corresponding reflection point. Make a list on the board for the class to see. Ask the class if they can see a pattern? If you give them a coordinate can they tell you the reflection coordinate?

Finally ask them to give you a rule: **For any point (a,b) the reflection of it over the line $y = x$ is (b, a) .**

Translations: Using the same location of the original as they had in the reflection lesson, have the students glue down another rectangle and record its vertices' coordinates.

Have the students place a copy of the rectangle on top of the original and instruct them to slide the copy two units to the right.

1. glue the copy down in its new position (this is the image),
2. record the coordinate of the new figure. Indicate this is a horizontal translation,
3. Discuss and answer the questions that pertain to the horizontal translation.
Make a list of students' original vertices and the images' vertices.
4. Ask the students, "If I give you a set of coordinates for a point, could you determine the coordinates of the image after the translation?" Give them a point and see if they can give you the coordinates of the image. **(a, b) becomes (a+2, b)**
5. Now ask them if you give them another point and a different horizontal translation (3 units to the left) could they give you the location of the image. **(a, b) becomes (a+c, b) where c is the number of units moved and + indicate to the right or (a,b) becomes (a-c, b) where c is the number of units and – indicate to the left. Or (a,b) becomes (a+c, b) where c is the number of units moved horizontally.**

Do this same activity for the vertical translation. For example, choose 3 units down.

(a,b) becomes (a,b+d) where d is the vertically translation

Diagonal translation:

State: "It is difficult to tell the length of a translation when we go diagonally across the grid. I want you to follow these directions."

1. I want you to make a point at (2, -3) *these are the combinations of the previous two translations.*
2. Connect the origin to (2,-3) with a line segment.
3. Put one end of the coffee stirrer at the origin (0, 0).
4. Cross the original rectangle with the stirrer (keep the other end on the origin).
5. Put a copy of the rectangle over the stirrer and the original rectangle.
6. Tape the copy to the stirrer.
7. Slide the end of the stirrer along the line segment until it gets to the point (2,-3).
8. Tape the image to its new location and record the vertices coordinates.

Collect original vertices and the corresponding image coordinates of the translation.

(a, b) becomes (a+2, b+-3)

In general: for a diagonal translation, (a,b) becomes (a+c, b+d) where c is the horizontal move and d is the vertical move.

Rotation: Paddy paper and a brass brad

90⁰ rotation counter clockwise / 270⁰ clockwise rotation

Place a piece of paddy paper over the grid so that it covers the origin and the original rectangle ABCD. Remember to place your figures on the grid at the same location that you did on both the reflection and the translation. You should have the same original vertices points.

1. With your brass brad carefully punch through the paddy paper and the grid at the origin.
2. Tape down a copy of the rectangle over the original on the paddy paper. Make a mark on the paddy paper that matches up with the positive x-axis. This will be used to determine when rotations of 90⁰ been completed.
3. Instruct the students to rotate the figure on the paddy paper 90⁰ counter clockwise. They need to line up the line they made on the paddy paper to the positive y-axis.
4. Note where the vertices of the figure are now and glue down a copy of the figure on the coordinate grid. *(They will have to carefully lift the paddy paper in order to glue down the copy of the figure.)*
5. Have the students name the corresponding vertices of the new position of the rectangle A'B'C'D'.
6. Now have the students move the figure on the paddy paper so that it is back on the original figure and rotate the paddy paper 270⁰ clockwise (line up the line on the paddy paper with the positive y-axis. Note that the rectangle is in the same position as the 90⁰ rotation. So a 90⁰ rotation counter clockwise gives the same result as a 270⁰ clockwise rotation.
7. Record the coordinates of the image of the rectangle.
8. Ask several groups to report the coordinates of the vertices of their original positions and the corresponding vertices of their images after the rotation. Record these so the class can see.

For the 90⁰ counter clockwise and the 270⁰ clockwise rotation the students should say the opposite value of the y-coordinate becomes the x-coordinate and the value of the x-coordinate becomes the y-coordinate.

(a, b) becomes (-b, a)

180⁰ rotation counter clockwise / 180⁰ clockwise rotation

1. Have the students move the paddy paper figure back on top of the original position.
2. Rotate the figure 180⁰ counter clockwise.
3. Follow directions 4-8 in the previous rotation but using the degrees for this rotation.

The students should note that the x- and y-coordinates are now opposites of the original coordinates. (a, b) becomes (-a, -b).

270⁰ counter clockwise /90⁰ clockwise rotation

Complete the directions for the previous rotations but for 270 cc and 90 c.

The result should be (a, b) becomes (-b, -a).

Complete the questions in the handout and discuss the outcome (patterns). Ask, “*What do you think will happen if we used another figure?*” Instruct the students to choose another figure, make a prediction of what the new coordinates will be, and perform the transformations.

All sheets in the students’ books do not have to be completed. Some students will need more practice than others. Use your judgment when assigning the other figures. Supply some of your own figures if you want. If students can not remember what transformation results in what action, encourage them to choose a figure and complete the transformations that are remaining in their books.

Explain: Students will complete the handout and describe what happens to the coordinate of a figure after each transformation.

Elaborate: The students will be able to give the coordinate for each transformation when given the original coordinates and the amount and direction of transformation without actually performing the transformation using a grid.

Student Participation: Students will model the transformation for the class, discuss the outcomes, and make predictions about the coordinates based on their explorations.

Evaluate: I have added an evaluation at the end of the students’ book. You may not want to put this in the students’ books. Instead you may want to give this to them after you have completed the unit or give parts of this after practicing each transformation.

Transformations

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Transformations

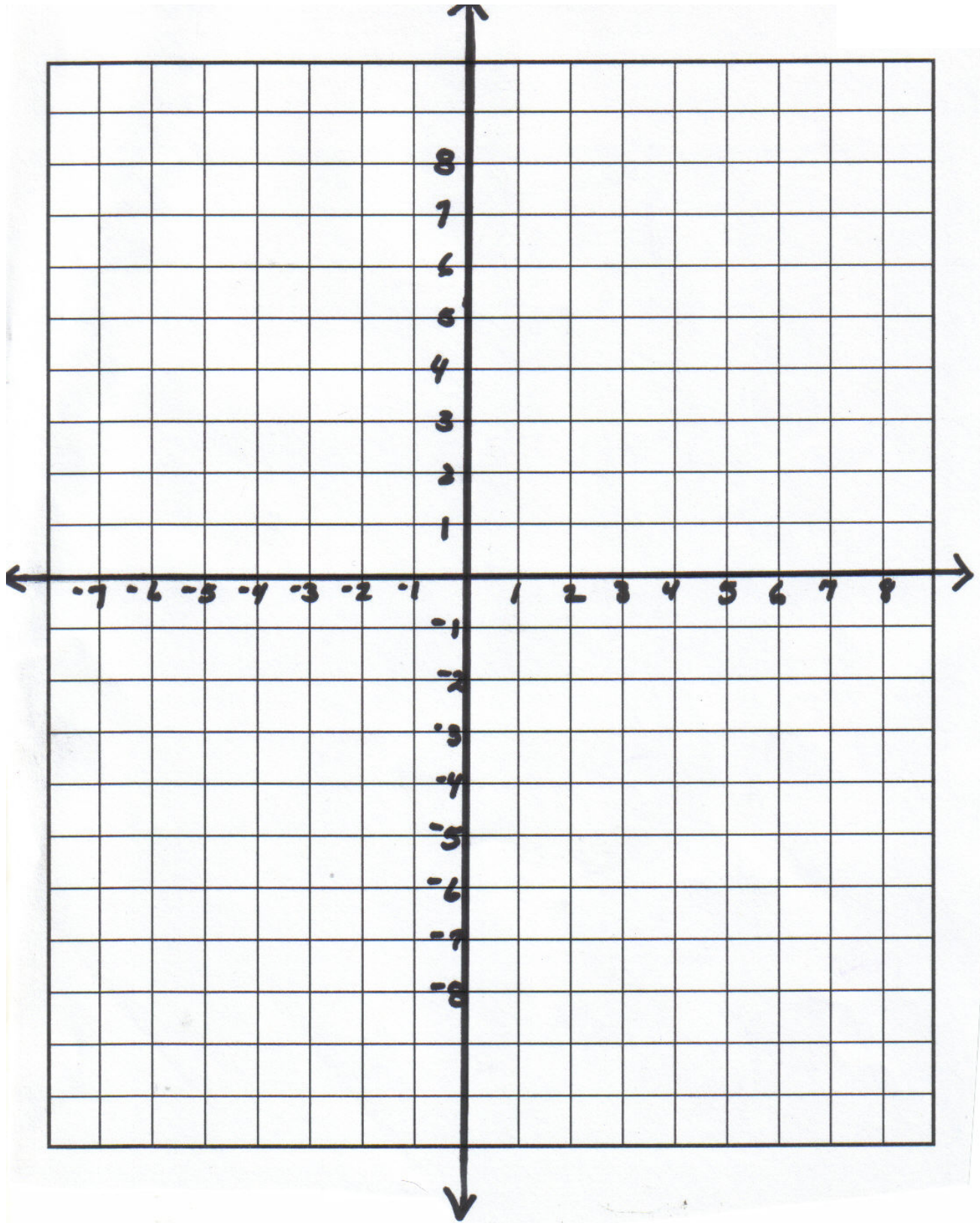
Transformations

Translation (Slide)

Rotation (Turn)

Reflection (Flip)

Name _____ Class _____



Shape	Original Coordinates	Reflection x-axis Coordinates	Reflection y-axis Coordinates	Reflection Line $y=x$
Rectangle	A:	A':	A'':	A''':
	B:	B':	B'':	B''':
	C:	C':	C'':	C''':
	D:	D':	D'':	D''':

What are the distances of the corresponding vertices of the figure and its image from the x-axis?

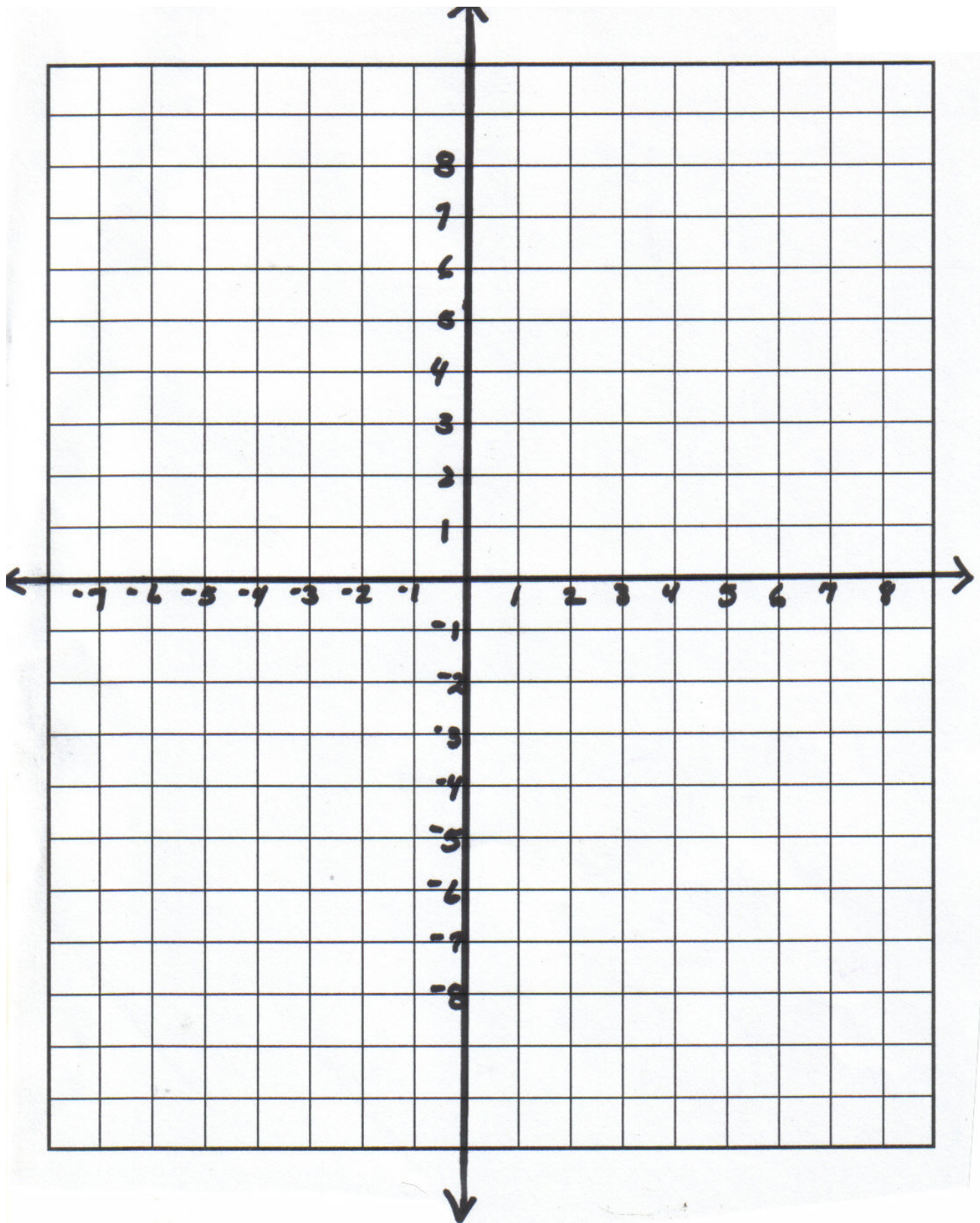
How to the corresponding vertices relate (What is the rule)? _____

What are the distances of the corresponding parts of the figure and its image from the y-axis?

How to the corresponding parts relate (What is the rule)? _____

How do the distances from the line $y = x$ of the corresponding parts of the figure and its reflections relate? _____

How do the corresponding parts relate (What is the rule)? _____



Shape	Original Coordinates	Horizontal Translation Coordinates	Vertical Translation Coordinates	Diagonal Translation Coordinates
Rectangle	A:	A':	A'':	A''':
	B:	B':	B'':	B''':
	C:	C':	C'':	C''':
	D:	D':	D'':	D''':

How do the coordinates of the original relate to the coordinates of the translations (What is the rule)?

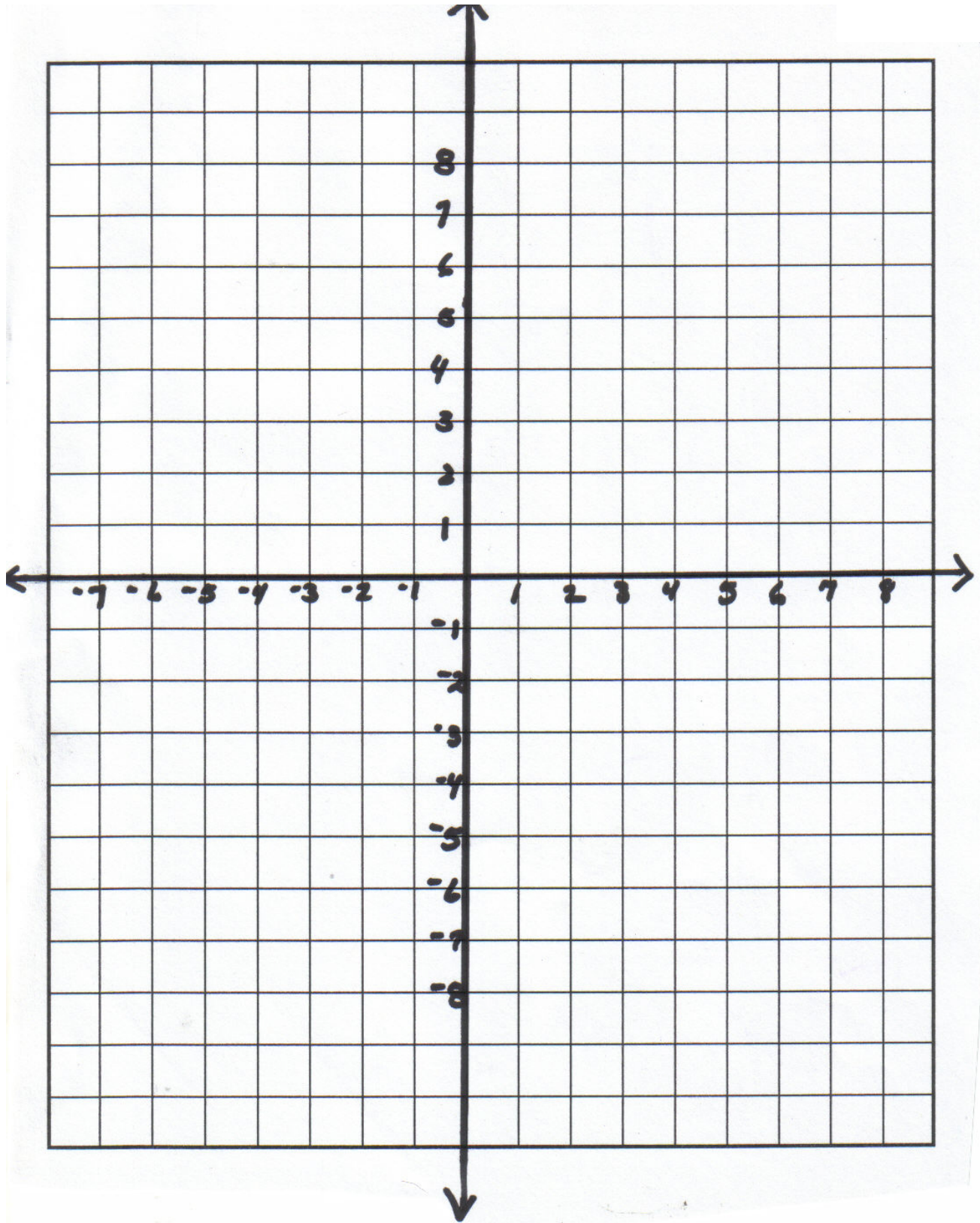
Horizontal: _____

Vertical: _____

Diagonal: _____

How are the transformations alike?

How are the transformations Different?

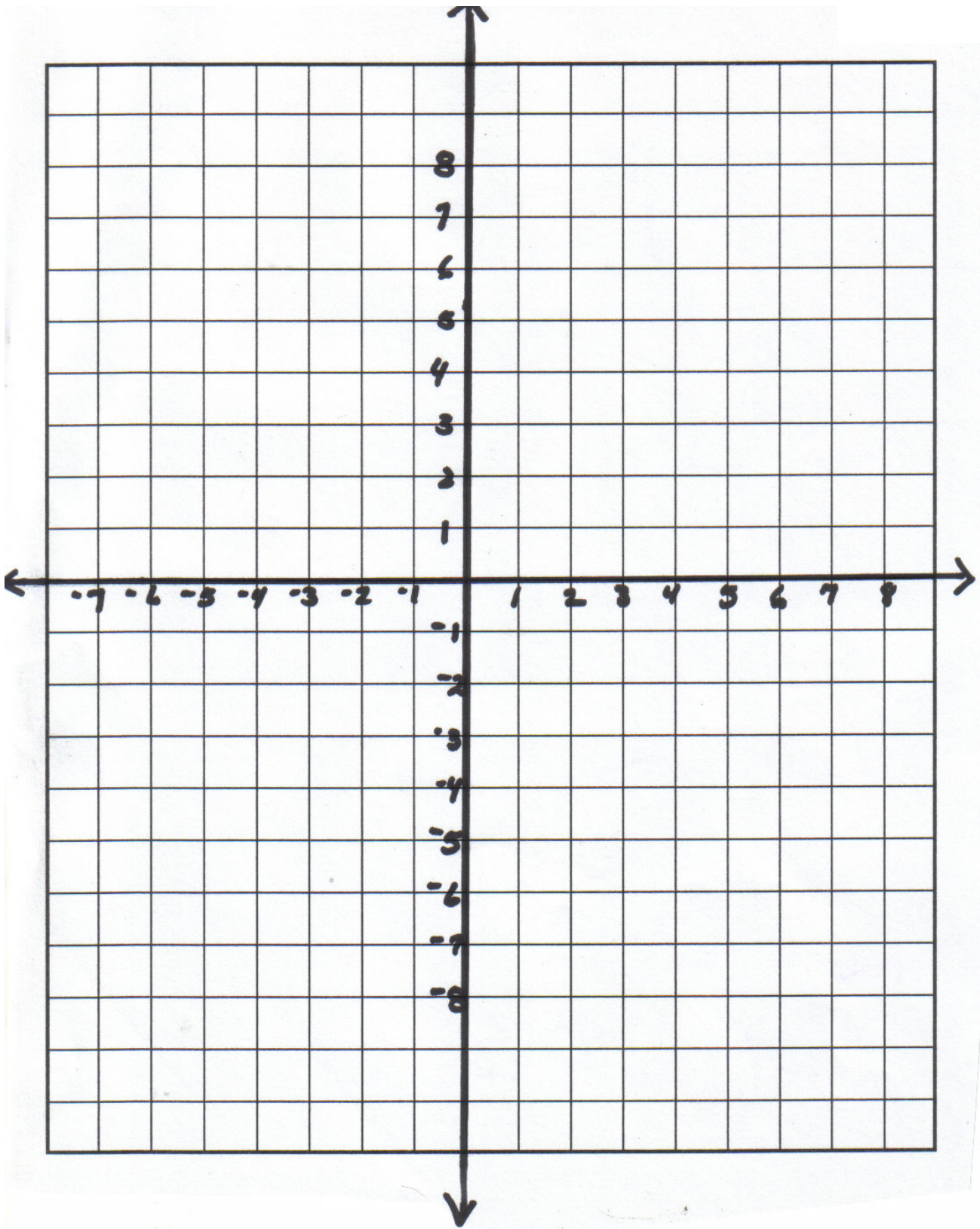


Shape	Original Coordinates	Rotation Coordinates $90^{\circ}\text{cc}/270^{\circ}\text{c}$	Rotation Coordinates $180^{\circ}\text{cc} \& \text{c}$	Rotational Coordinates $270^{\circ}\text{cc}/90^{\circ}\text{c}$
Rectangle	A: B: C: D:	A': B': C': D':	A'': B'': C'': D'':	A''': B''': C''': D''':

How do the corresponding parts of a 90° counter clockwise / 270° clockwise rotation relate (what is the rule)?

How do the corresponding parts of a 180° counter clockwise / 180° clockwise rotation relate (what is the rule)?

How do the corresponding parts of a 270° counter clockwise / 90° clockwise rotation relate (what is the rule)?



Shape	Original Coordinates	Reflection x-axis Coordinates	Reflection y-axis Coordinates	Reflection Line $y=x$
Triangle	E: F: G:	E': F': G':	E'': F'': G'':	E''': F''': G''':

What are the distances of the corresponding vertices of the figure and its image from the x-axis?

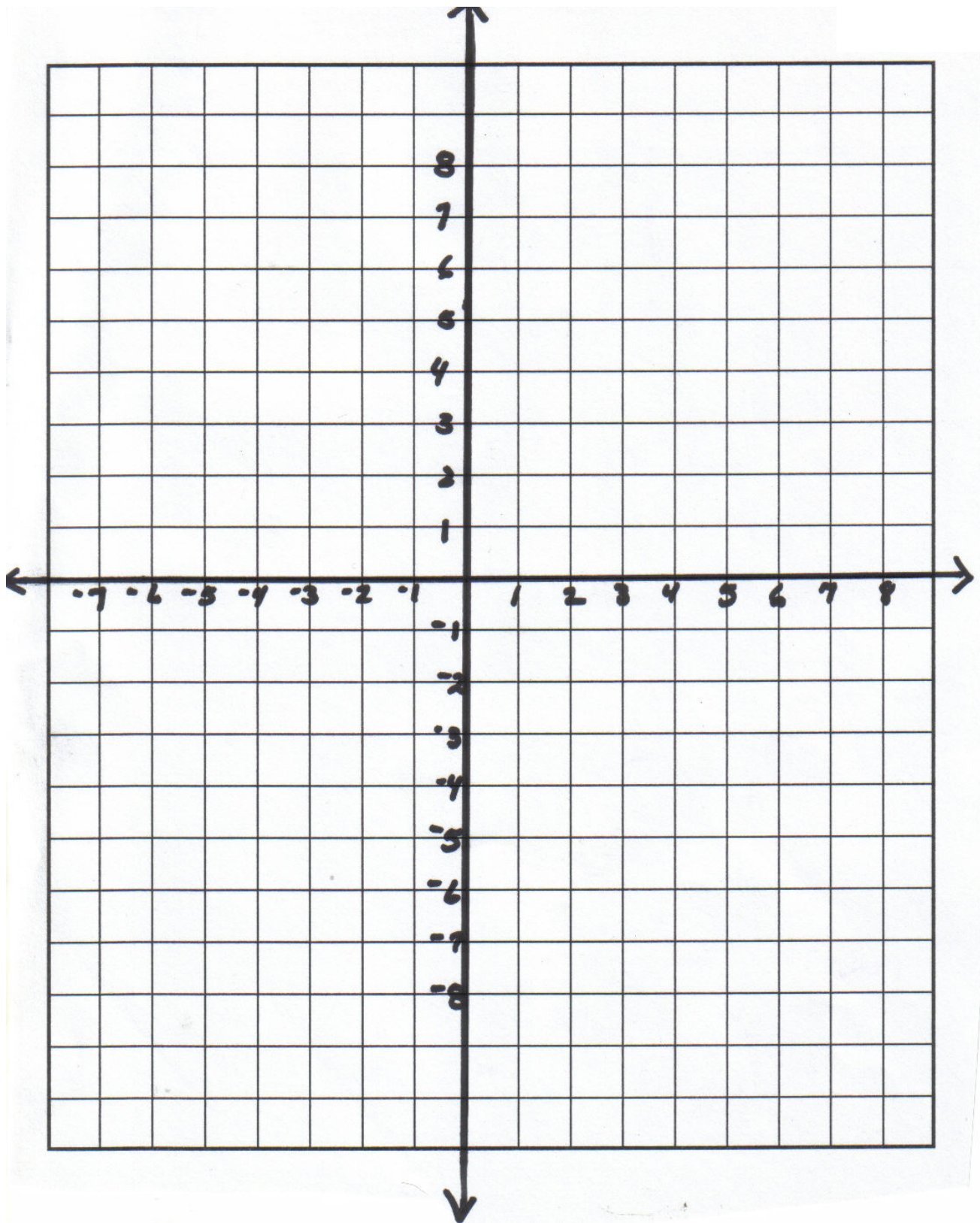
How to the corresponding vertices relate (What is the rule?)? _____

What are the distances of the corresponding parts of the figure and its image from the y-axis?

How to the corresponding parts relate (What is the rule?)? _____

How do the distances from the line $y = x$ of the corresponding parts of the figure and its reflections relate? _____

How do the corresponding parts relate (What is the rule?)? _____



Shape	Original Coordinates	Horizontal Translation Coordinates	Vertical Translation Coordinates	Diagonal Translation Coordinates
Triangle	E: F: G:	E': F': G':	E'': F'': G'':	E''': F''': G''':

How do the coordinates of the original relate to the coordinates of the translations (What is the rule)?

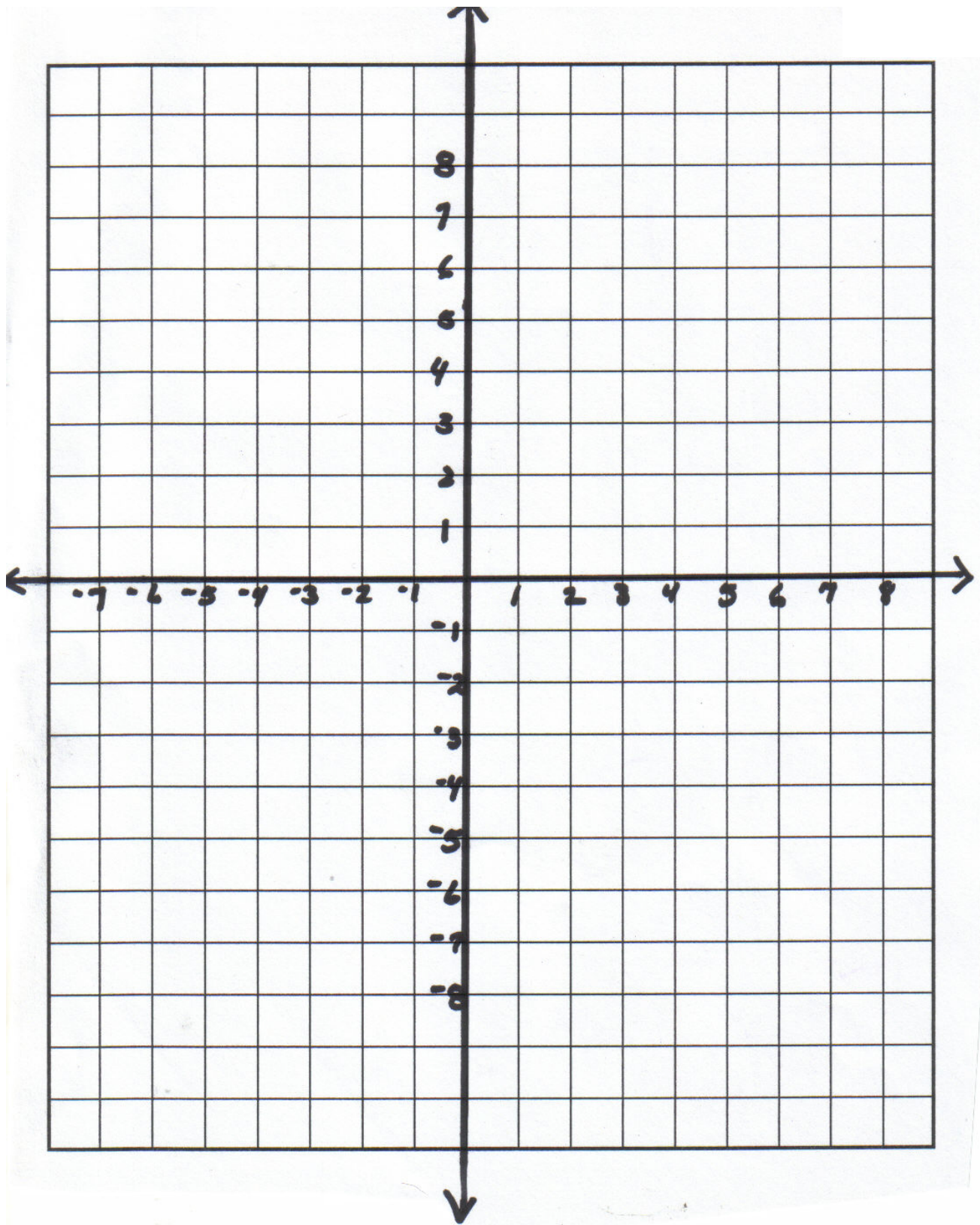
Horizontal: _____

Vertical: _____

Diagonal: _____

How are the transformations alike?

How are the transformations Different?

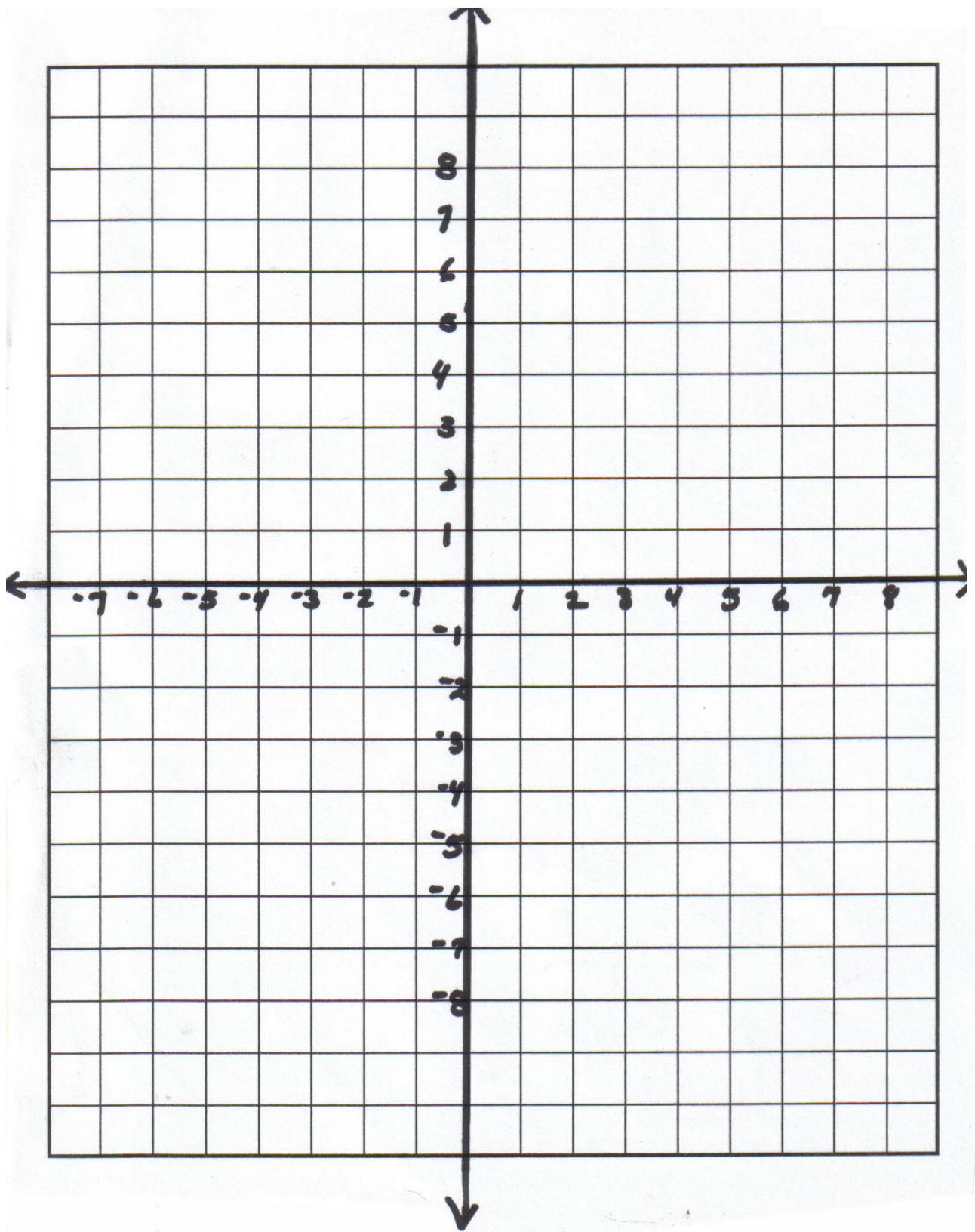


Shape	Original Coordinates	Rotation Coordinates $90^{\circ}\text{cc}/270^{\circ}\text{c}$	Rotation Coordinates $180^{\circ}\text{cc} \& \text{c}$	Rotational Coordinates $270^{\circ}\text{cc}/90^{\circ}\text{c}$
Triangle	E: F: G:	E': F': G':	E'': F'': G'':	E''': F''': G''':

How do the corresponding parts of a 90° counter clockwise / 270° clockwise rotation relate (what is the rule)?

How do the corresponding parts of a 180° counter clockwise / 180° clockwise rotation relate (what is the rule)?

How do the corresponding parts of a 270° counter clockwise / 90° clockwise rotation relate (what is the rule)?



Shape	Original Coordinates	Reflection x-axis Coordinates	Reflection y-axis Coordinates	Reflection Line $y=x$
Trapezoid	H: I: J: K:	H': I': J': K':	H'': I'': J'': K'':	H''': I''': J''': K''':

What are the distances of the corresponding vertices of the figure and its image from the x-axis?

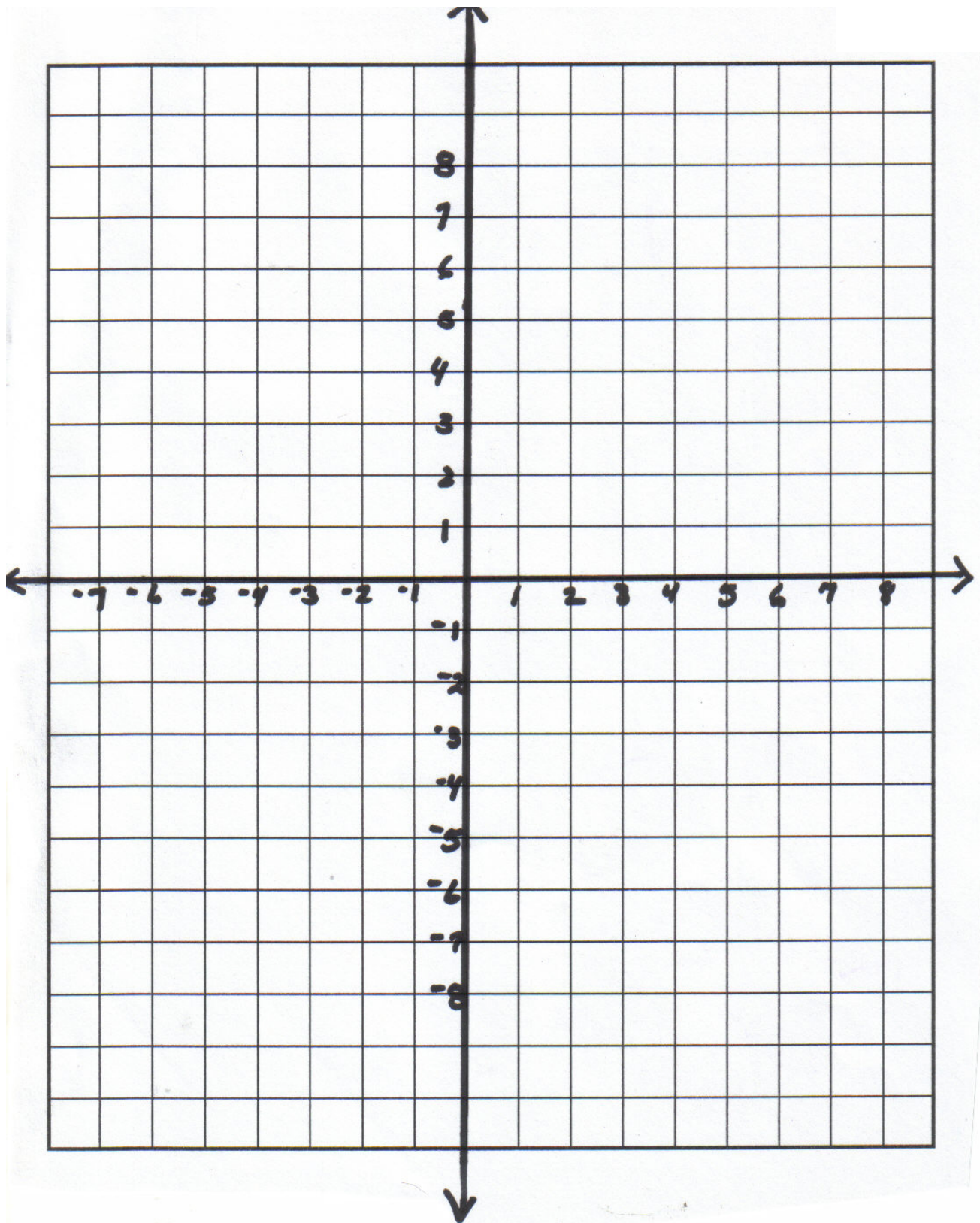
How to the corresponding vertices relate (What is the rule)? _____

What are the distances of the corresponding parts of the figure and its image from the y-axis?

How to the corresponding parts relate (What is the rule)? _____

How do the distances from the line $y = x$ of the corresponding parts of the figure and its reflections relate? _____

How do the corresponding parts relate (What is the rule)? _____



Shape	Original Coordinates	Horizontal Translation Coordinates	Vertical Translation Coordinates	Diagonal Translation Coordinates
Trapezoid	H: I: J: K:	H': I': J': K':	H'': I'': J'': K'':	H''': I''': J''': K''':

How do the coordinates of the original relate to the coordinates of the translations (What is the rule)?

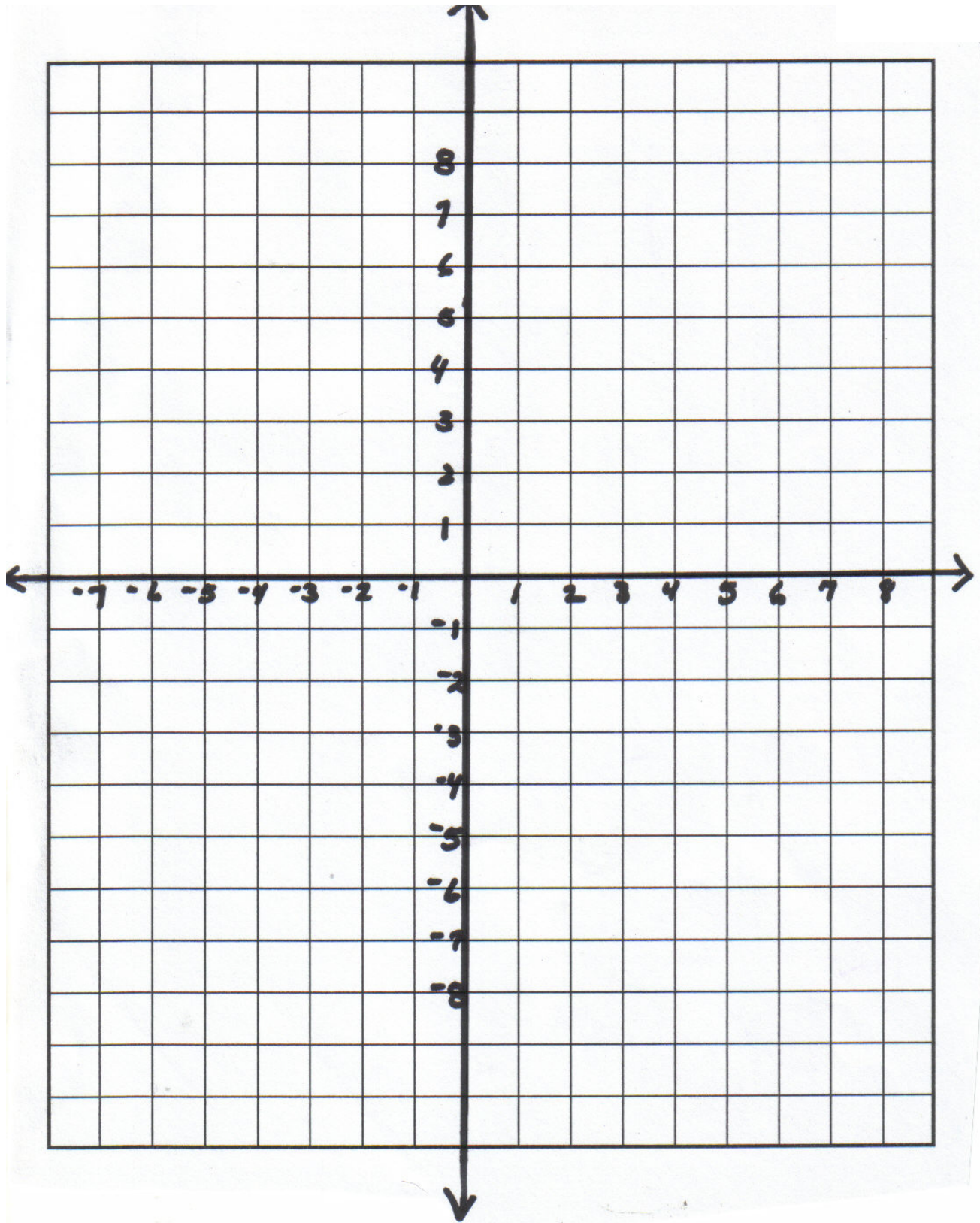
Horizontal: _____

Vertical: _____

Diagonal: _____

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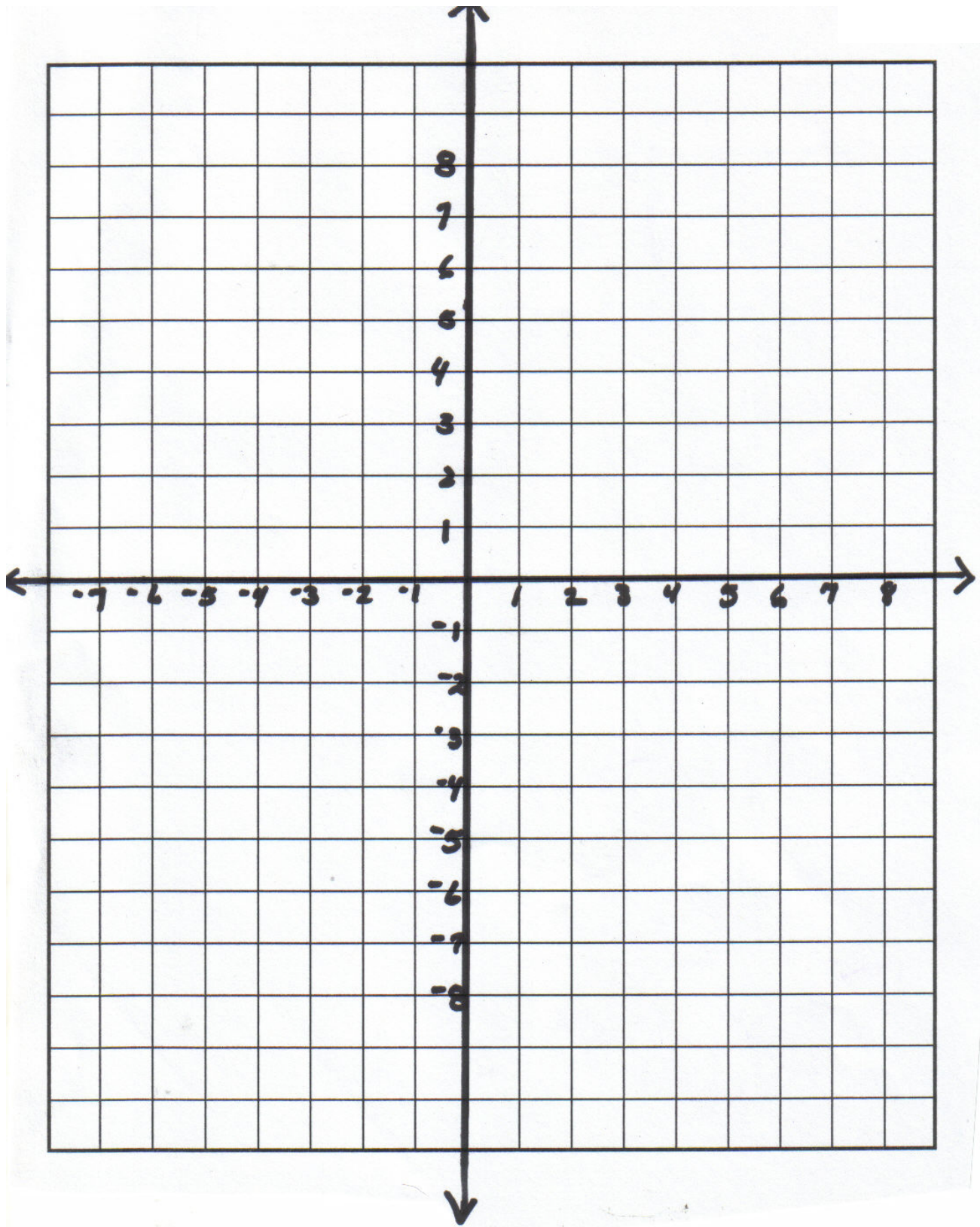


Shape	Original Coordinates	Rotation Coordinates $90^{\circ}\text{cc}/270^{\circ}\text{c}$	Rotation Coordinates $180^{\circ}\text{cc} \& \text{c}$	Rotational Coordinates $270^{\circ}\text{cc}/90^{\circ}\text{c}$
Trapezoid	H: I: J: K:	H': I': J': K':	H'': I'': J'': K'':	H''': I''': J''': K''':

How do the corresponding parts of a 90° counter clockwise / 270° clockwise rotation relate (what is the rule)?

How do the corresponding parts of a 180° counter clockwise / 180° clockwise rotation relate (what is the rule)?

How do the corresponding parts of a 270° counter clockwise / 90° clockwise rotation relate (what is the rule)?



Shape	Original Coordinates	Reflection x-axis Coordinates	Reflection y-axis Coordinates	Reflection Line $y=x$
Triangle	L: M: N:	L': M': N':	L'': M'': N'':	L''': M''': N''':

What are the distances of the corresponding vertices of the figure and its image from the x-axis?

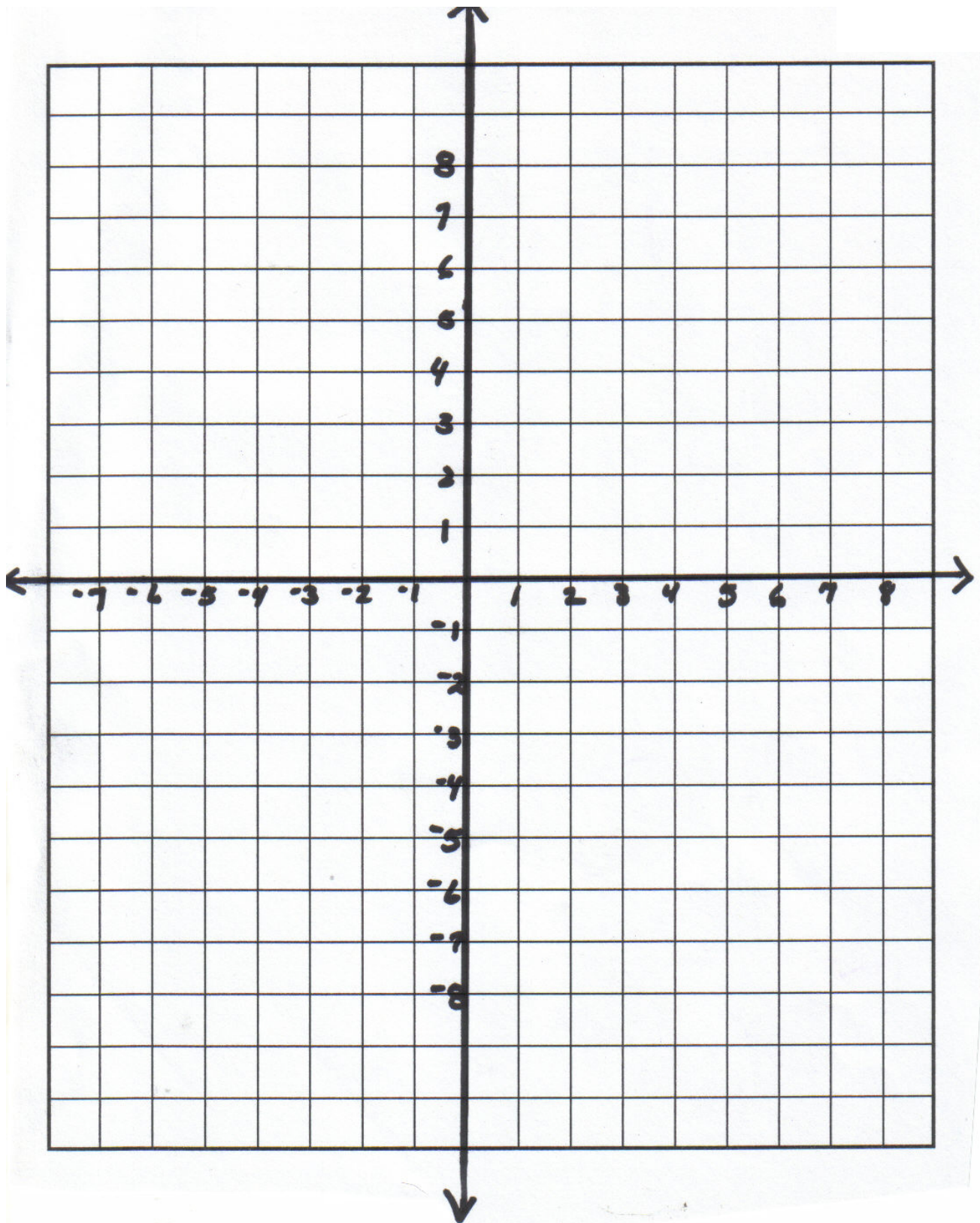
How to the corresponding vertices relate (What is the rule?)? _____

What are the distances of the corresponding parts of the figure and its image from the y-axis?

How to the corresponding parts relate (What is the rule?)? _____

How do the distances from the line $y = x$ of the corresponding parts of the figure and its reflections relate? _____

How do the corresponding parts relate (What is the rule?)? _____



Shape	Original Coordinates	Horizontal Translation Coordinates	Vertical Translation Coordinates	Diagonal Translation Coordinates
Triangle	L: M: N:	L': M': N':	L'': M'': N'':	L''': M''': N''':

How do the coordinates of the original relate to the coordinates of the translations (What is the rule)?

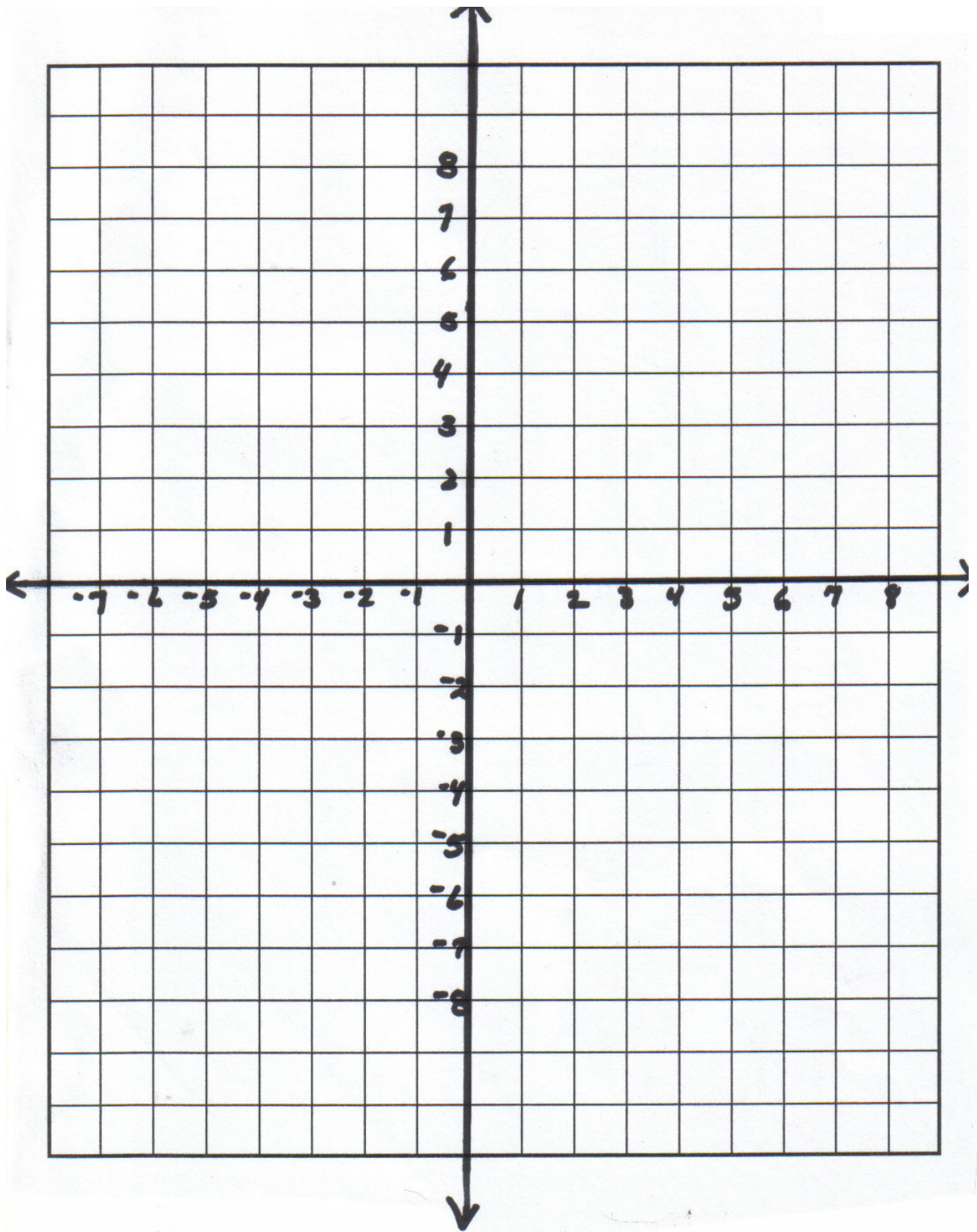
Horizontal: _____

Vertical: _____

Diagonal: _____

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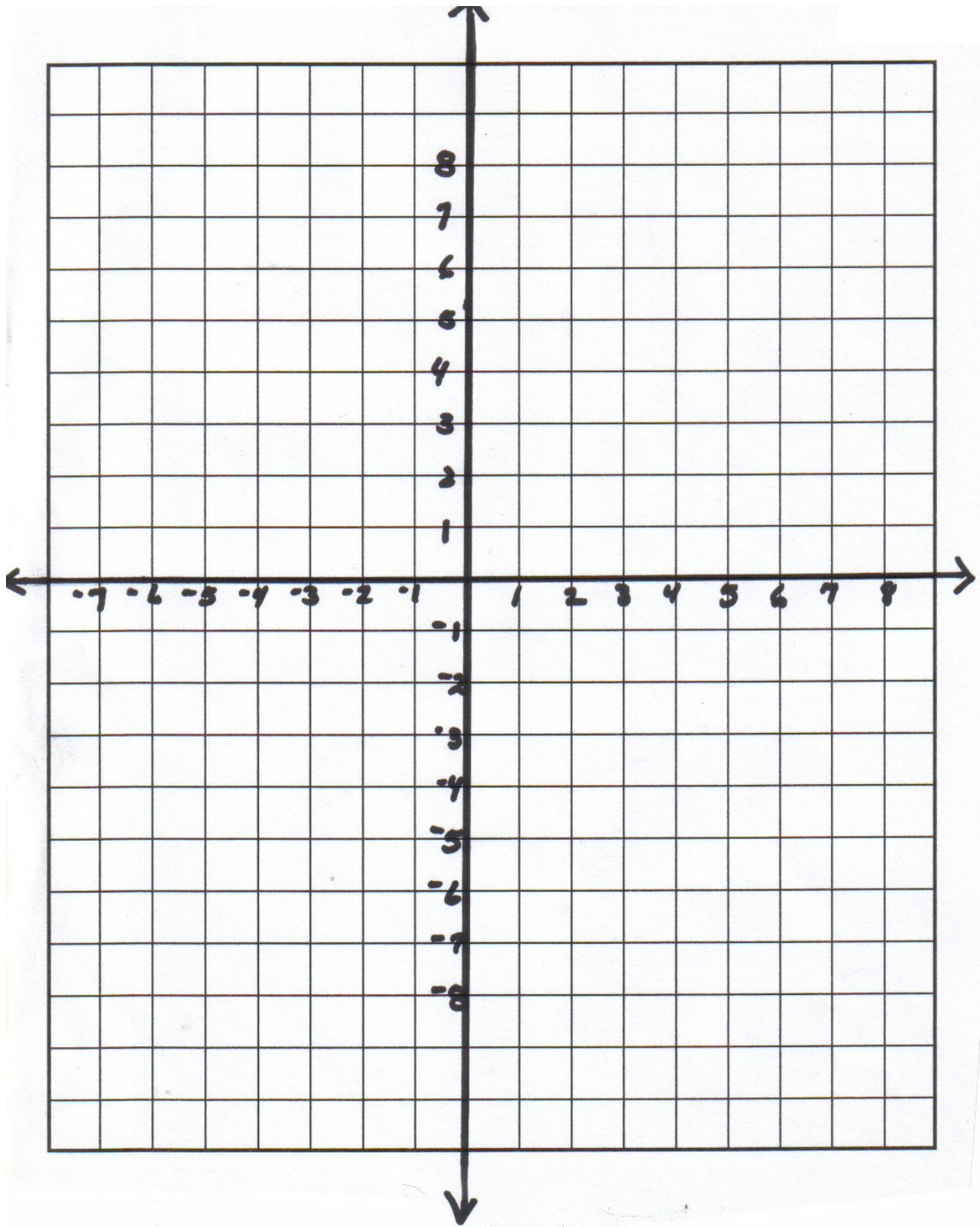


Shape	Original Coordinates	Rotation Coordinates $90^{\circ}\text{cc}/270^{\circ}\text{c}$	Rotation Coordinates $180^{\circ}\text{cc} \& \text{c}$	Rotational Coordinates $270^{\circ}\text{cc}/90^{\circ}\text{c}$
Triangle	L: M: N:	L': M': N':	L'': M'': N'':	L''': M''': N''':

How do the corresponding parts of a 90° counter clockwise / 270° clockwise rotation relate (what is the rule)?

How do the corresponding parts of a 180° counter clockwise / 180° clockwise rotation relate (what is the rule)?

How do the corresponding parts of a 270° counter clockwise / 90° clockwise rotation relate (what is the rule)?



Shape	Original Coordinates	Reflection x-axis Coordinates	Reflection y-axis Coordinates	Reflection Line $y=x$

What are the distances of the corresponding vertices of the figure and its image from the x-axis?

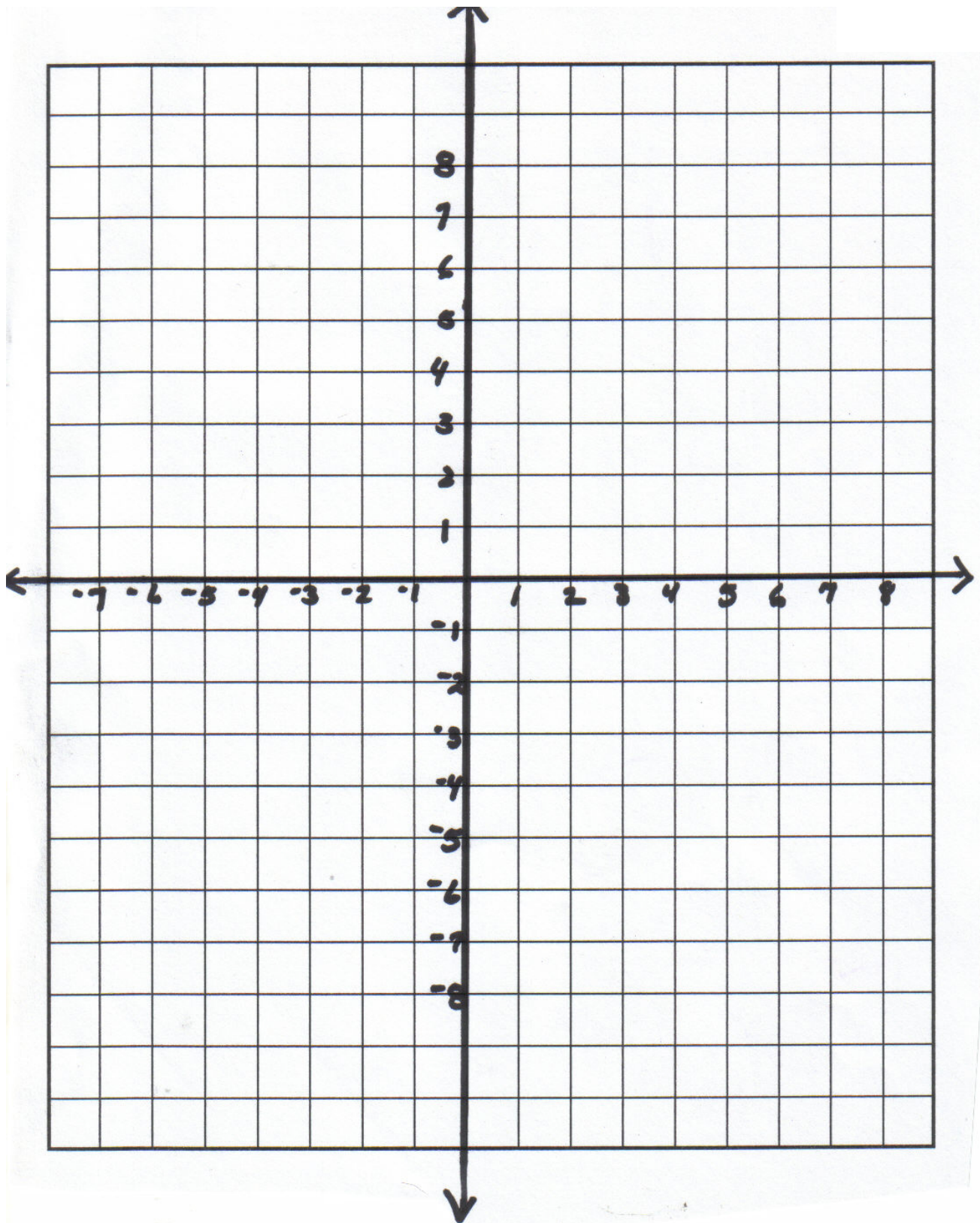
How to the corresponding vertices relate (What is the rule?)? _____

What are the distances of the corresponding parts of the figure and its image from the y-axis?

How to the corresponding parts relate (What is the rule?)? _____

How do the distances from the line $y = x$ of the corresponding parts of the figure and its reflections relate? _____

How do the corresponding parts relate (What is the rule?)? _____



Shape	Original Coordinates	Horizontal Translation Coordinates	Vertical Translation Coordinates	Diagonal Translation Coordinates

How do the coordinates of the original relate to the coordinates of the translations (What is the rule)?

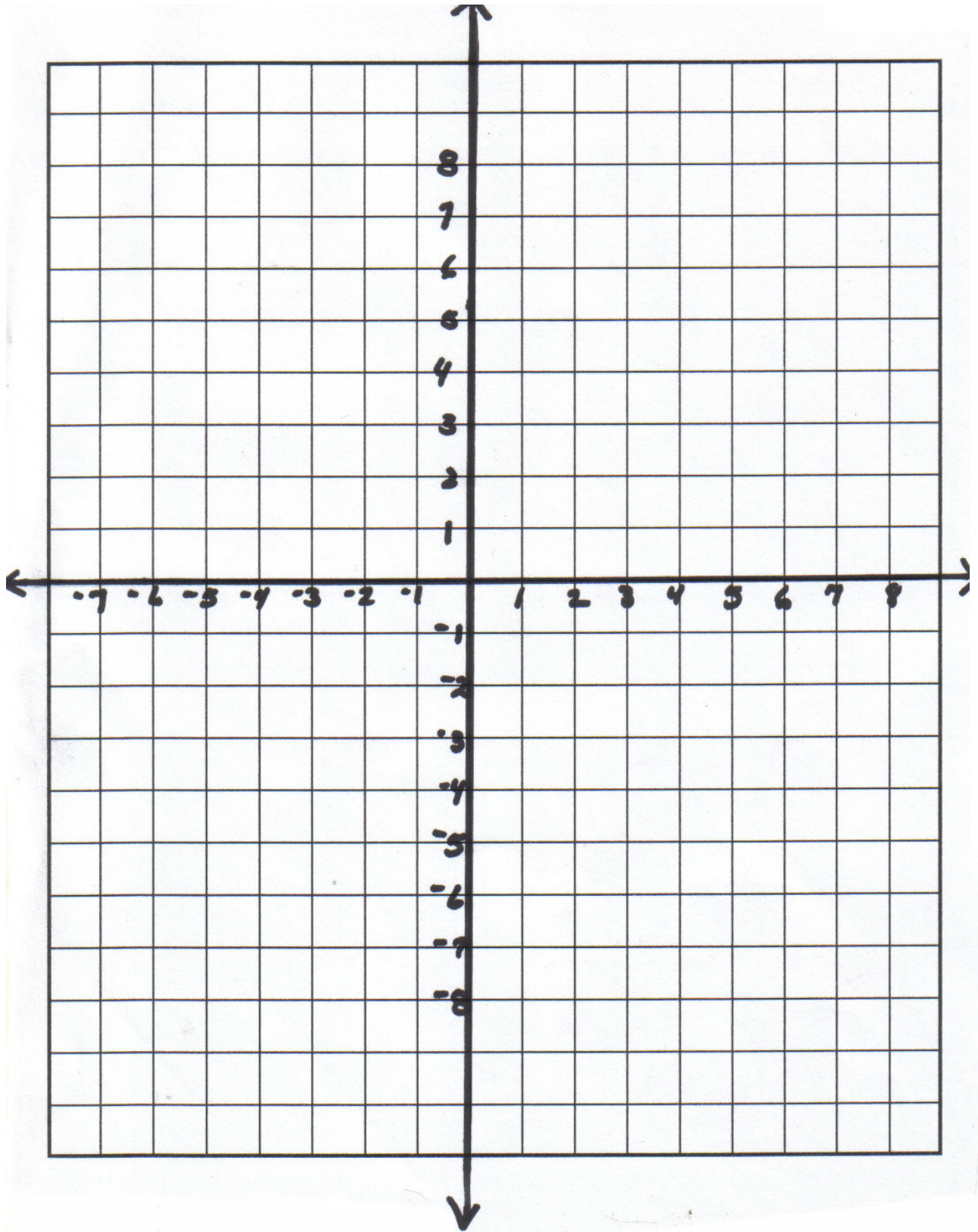
Horizontal: _____

Vertical: _____

Diagonal: _____

How are the transformations alike?

How are the transformations Different?



Shape	Original Coordinates	Rotation Coordinates 90° cc/ 270° c	Rotation Coordinates 180° cc & c	Rotational Coordinates 270° cc/ 90° c

How do the corresponding parts of a 90° counter clockwise / 270° clockwise rotation relate (what is the rule)?

How do the corresponding parts of a 180° counter clockwise / 180° clockwise rotation relate (what is the rule)?

How do the corresponding parts of a 270° counter clockwise / 90° clockwise rotation relate (what is the rule)?

The coordinate of WXYZ are listed below. Without performing the actual transformations, state the coordinates for the images' vertices.

Shape	Original Coord.	Horizontal Translation 4 units down Coordinates	Vertical Translation 2 units left Coordinates	Translation of 2 units up and 4 units right Coordinates	Rotation 90° Cc Coordinates	Rotation 180° Clockwise Coordinates	Reflection x-axis Coordinates	Reflection y-axis Coordinates	Reflection $y = x$ Coordinates
Rectangle	W: (4, 5)	W':	W'':	W''':	W'''':	W''''':	W'''''':	W''''''':	W'''''''':
	X: (8, 5)	X':	X'':	X''':	X'''':	X''''':	X'''''':	X''''''':	X'''''''':
	Y: (8, 2)	Y':	Y'':	Y''':	Y'''':	Y''''':	Y'''''':	Y''''''':	Y'''''''':
	Z: (4, 2)	Z':	Z'':	Z''':	Z'''':	Z''''':	Z'''''':	Z''''''':	Z'''''''':